**Integral Calculus**

Def: 1. **Integration is the inverse of differentiation**

2**. Integration is summation**

**Two types of integral**

1. **Indefinite Integral**
2. **Definite Integral**

**Indefinite Integral**

**FUNDAMENTAL FORMULAS**

1. **,**
2. **,**
3. **,**
4. **,**
5. **,**
6. **,**
7. **,**
8. **,**
9. **,**
10. **,**
11. **,**
12. **,**

**Methods of Integration**

1. Integration by substitution
2. Integration by parts
3. Integration by partial fraction
4. Integration by successive reduction

**Integration by substitution**

**1Example:** Workout

Solution:

put

**2Example:** Workout

Solution:

Put

**Example:** Workout

Solution: put ,

== 6

=6

**=**6+6

=6(+-z)+ 6 tanz +c where

**2Example:** Workout

Solution:

Put

**Type-1**

**Procedure Substitute**

**3Example:** Workout

Solution:

put

**Type-2**

**Procedure Substitute**

**4Example:** Workout

Solution:  **,** here

put

:

**Type-3 ,** n is +ve even integer

**Procedure Substitute**

**5Example:** Workout

Solution:

put

:

**1111111111111111111**

**Standard integral formula**

1. =+c,
2. =+c,
3. =+c ,

**Type-4**

**Procedure**

**Example:** Workout

Solution: =

= =

**Example:** Workout

Solution: =

= =

**Type-5**

**Procedure**

**Example:** Workout

Solution: =

=+

=+=lnz++C

**FORMULA**

1. ln(secx)+c
2. ln(sinx)+c
3. ln (secx+tan x)+c
4. ln tan(x/2 )+c
5. =ln(x+ +c, x=atan,
6.  =ln(x+)+c , x=asec,
7. =+c x=asin,

 x=atan dx=d

==ln(sec+tan)+c= ln(x+)+c

**Type**6 

**Procedure :**

**Example:** Workout 

Solution: ==

==ln+c

**Example:** Workout 

Solution: =

=

==sin+C

**Type-7** 

**Procedure s**ubstitute

**Example:** Workout 

Solution: put , dx=2zdz

==2 =2ln(z+)+c

=2ln()+c

=2ln( )+c

**Type-8**

**Procedure**

**Example:** Workout 

Solution: I ==

=+

,

 put z=

= = = 

==

==ln

**Type**-9

Process

**Example:** Workout

Solution: =

**Type- 10**

**Procedure**  Substitute

**Example:** Workout 

Solution put dx=2zdz

===+c=+c

**Type11**

**Procedure**  Substitute 

**Example:** Workout

Solution:

put 1+x, , dx

====

=

= =

=+c

**Type12**  ,

**Procedure**  Substitute ,

**Example:** Workout

Solution:

put ,

**2222222222222**

**FORMULA**

1. = ln(x+) x=atan, acot

8. = ln(x+)x=asec, acosec

9. = x=asin, acos

The results can be obtained by two methods

Integration by substitution and Integration by parts

I==-

=- =-

=-+

2I=+

I= + ln(x+)+c

Type 13 dx

**Procedure**  dx

**Example:** Workout

Solution: =

=

=+c

**Type 14**

**Procedure**

**Example:** Workout

Solution :

=  +

=+

=+

= 

++c

**Type15**  , ,

**Procedure**  Substitute

Examples , , ,

**Example:** Workout

Solution

= , put , 

==

==

=

**Example:** Workout 

Solution

Put

=

put , 

===

==

=

Type16 , ,

,

**Procedure**  Substitute

find the value of *l , m & n*

Examples Workout , ,

,

Exam Workout

**Solution:**

Let

Equating the coefficient of *cosx*  & *sinx,* we get

, from this eqution we get ,

Exam Workout

**Solution:**

Let

+n

Equating the coefficient of *cosx*  & *sinx,* we get

,

from this eqution we get

**Integration by parts**

**FORMULA:**

Ex.

= =

Ex.

=2[] =2+c

Exam Workout

Solu , put ,

**Formula**

1. +c

**Example:** Workout 

Sol. =

Let

Ans

**Example:** Workout

Sol.

Let

Ans

**Example:** Workout

Sol.

Ans

Exam Workout

Solu , put ,

+c

+c

+c

**Integration by partial fraction**

**Example:** Workout 

=+

1=A(x-2)+B(x-1) Putting x=1 A=-1 x=2 B=1

=-

=-=ln(x-2)-ln(x-1)+c

**Example:** Workout 

=++

=++



=ln(x-a) + ln(x-b) + ln(x-c)+c

**Example:** Workout 

= +

=tan+tan+c



=

**Type**  put 

**Example:** Workout



== = ==-3+3-=--3lnz+3z-+c, 

**Example:** Workout ==

**Example:** Workout 

==

=

**Example:** Workout dx==

===

**Type**

i) m or n is odd

ii) m and n is even

iii) m+n is an even negative integer

Examples

i) , , , , 

ii) , , 

iii), 

**Example:** Workout 

==

**Example:** Workout==

===

= 2cosAcosB=cos(A+B)+ cos(A-B)

=

**Alternative method**

**Example:** Workout 

By De Moivre’s theorem is 

let 



,,,

==

==-2-+4

=2cos6x-2.2cos4x-2cos2x+4

=(cos6x-2cos4x-cos2x+2)dx

 do

**Example:** Workout 

=

=

**Integration by successive reduction**

**Example:** Find the reduction formula for  hence find 



=

=

=





=

=()

=

=()

=

**Example:** Find the reduction formula for  hence find 



=

=

=-

=





=

= 

=

**Example:** Find the reduction formula for  hence find 







**Example:** Find the reduction formula for  hence find 



=

=











 Find 

**Example:** Find the reduction formula for  hence find 





**Example:** Find the reduction formula for 





=

=







**Example:** Find the reduction formula for 









**Example:** Find the reduction formula for hence find



 cos(nx-x)=cosnx cosx+sinnx sinx  sinnx sinx =cos(n-1)x-cosnxcosx 











